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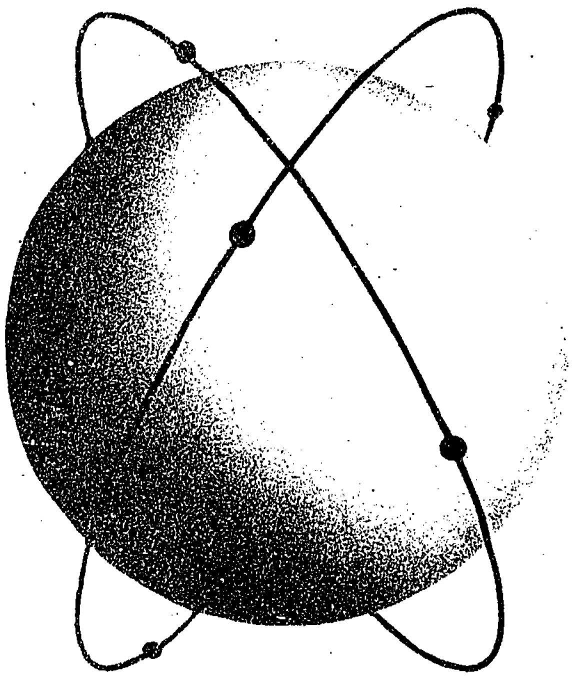
ABSTRACT

As the third lesson of the Articulated Multimedia Physics Course, instructional materials are presented in this study guide. An introductory description is given for scientific notation methods. The subject content is provided in scrambled form, and the use of matrix transparencies is required for students to control their learning process. Students are asked to use a magnetic tape playback, instructional tapes, and single concept films at the appropriate place in conjunction with a worksheet. Included are a problem assignment sheet and a study guide slipsheet. Related documents are SE 015 963 through SE 015 977. (CC)

ARTICULATED MULTIMEDIA PHYSICS



LESSON

3

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

NEW YORK INSTITUTE OF TECHNOLOGY

Old Westbury, Long Island

New York, N.Y.

ARTICULATED MULTIMEDIA PHYSICS

Lesson Number 3

THE ARITHMETIC OF SCIENTIFIC NOTATION

IMPORTANT: Your attention is again called to the fact that this is not an ordinary book. It's pages are scrambled in such a way that it cannot be read or studied by turning the pages in the ordinary sequence. To serve properly as the guiding element in the Articulated Multimedia Physics Course, this Study Guide must be used in conjunction with a Program Control equipped with the appropriate matrix transparency for this Lesson. In addition, every Lesson requires the availability of a magnetic tape playback and the appropriate cartridge of instructional tape to be used, as signaled by the Study Guide, in conjunction with the Worksheets that appear in the blue appendix section at the end of the book. Many of the lesson Study Guides also call for viewing a single concept film at an indicated place in the work. These films are individually viewed by the student using a special projector and screen; arrangements are made and instructions are given for synchronizing the tape playback and the film in each case.

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New York Institute of Technology
Articulated Multimedia Physics

LESSON 3

STUDY GUIDE SLIPSHEET

Please made the corrections indicated below before starting on Lesson 3.

STUDY GUIDE TEXT: Page 83, numerator of fraction at the top of the page. Change 3.54×10^4 to 3.56×10^4.

STUDY GUIDE DIAGRAMS: No corrections.

WORKSHEETS: Page 112, Question 5. Note that the "x" in the each of the choices stands for a multiplication sign, not an "unknown".

Page 112, Question 6. Cross out: $(4.20 \times 10^5)^3$ given in the statement of the problem. Substitute by writing in $(4.20 \times 10^4)^2$.

HOMEWORK PROBLEMS: Pink sheet, last page in book. Write in the bottom margin "Next page, please".

The previous lesson taught you how to write numbers in scientific notation. You should now be able to recognize that scientific notation methods have some important advantages over conventional number-writing.

First, scientific notation permits you to write both large and small numbers with a minumum of effort. Look at the number 127,000 billion billion billion in conventional notation:

127,000,000,000,000,000,000,000,000,000,000

In scientific notation it is simply: 1.27×10^{32}.

Second, you can show the number of significant figures in a measurement, or in a calculation from measurements, much more definitely when you use scientific notation. If someone tells you that a certain sphere has a volume of 500,000 cm^3, there is no way to determine whether this measurement has 1, 2, 3, 4, or 5 significant figures. But suppose it is written this way:

$$5.0 \times 10^5 \ cm^3$$

Now you know immediately that the measurement is correct to 2 significant figures.

Please go on to page 2.

Third, many problems involving arithmetic are greatly simplified by methods of scientific notation. In this lesson we shall deal with the various operations of arithmetic--multiplication, division, addition, subtraction, etc.--that you will meet throughout your physics course, from the point of view of scientific notation.

Like so many new techniques, these processes may at first appear more laborious than those of conventional arithmetic. But as you proceed to gain increasing facility with scientific notation you will find it reduces the amount of work needed, and insures greater accuracy by its simplicity.

Before proceeding to the next page, please turn to page 111 in the blue appendix.

We'll start with a quick review of the technique of writing numbers in scientific notation. Check the tabulation below; be sure to go over each one carefully to be certain that you understand it.

$1{,}301 = 1.301 \times 10^3$
$156.80 = 1.5680 \times 10^2$
$0.0065 = 6.5 \times 10^{-3}$
$0.000723 = 7.23 \times 10^{-4}$
$1{,}750{,}000 = 1.750 \times 10^6$ (4 significant figures)
$1{,}750{,}000 = 1.75 \times 10^6$ (3 significant figures)

Any number written in scientific notation consists of two parts. In the number 6.5×10^{-3}, for example, the portion "10^{-3}" is called the power of ten. What is the portion "6.5" called?

(1)

A The main number.

B The exponent.

C The coefficient.

YOUR ANSWER --- A

Your selection was wrong. There is nothing wrong with this solution. Here--check it:

(1) -4.8 divided by 6.0 is an example of a negative number divided by a positive number to yield a negative quotient. Thus, the quotient is -0.8. We will later change this to -8.0 by moving the decimal one place to the right.

(2) 10^{-7} divided by 10^4 is handled by subtracting 4 from -7. But subtraction calls for a change of sign from 4 to -4, followed by algebraic addition: $-7 + -4 = -11$.

(3) Thus far we have, then: -0.8×10^{-11}

(4) Now change this to proper scientific notation where the decimal is placed after the first non-zero digit. In moving the decimal 1 place to the right (-0.8 to -8.0), we have multiplied the coefficient by 10, hence we must compensate by dividing the exponent by 10. 10^{-11} divided by 10^1 is 10^{-12}, giving us finally:

$$-8.0 \times 10^{-12}$$

So you see that this problem was properly solved.

Please return to page 87 and go on with your solutions.

YOUR ANSWER --- B

You were confused by a big word. We round back coefficients not exponents.

Choosing this answer indicates your notebook is not well kept; and, it demonstrates that you're not thinking clearly about the answer you select because the statement in itself is in error.

Please return to page 88 and select another answer.

YOUR ANSWER --- D

You are correct. Good. This was your procedure: (1) You inspected the number and found that the exponent was odd. (2) You then shifted the decimal in the coefficient one place to the right and divided the power of ten by 10 to obtain $\sqrt{81.00 \times 10^{10}}$. (3) Then you wrote the square root of 81.00 as 9.000 to retain 4 significant figures, and divided the exponent by 2 to obtain the new power of ten, 10^5.

Please go on to page 7.

You won't be running into negative coefficients involving square roots in your physics course, so we shall ignore these. But you may very well encounter negative exponents. For example, the number 0.000049 when written in scientific notation has a negative exponent:

$$0.000049 = 4.9 \times 10^{-5}$$

You can find the square root of this number in exactly the same way as you did for numbers having positive exponents, provided that you observed the algebraic sign rules as always. Just remember, when a negative number is divided by 2, the quotient is negative. So, for the problem above, we have:

$$\sqrt{4.9 \times 10^{-5}} = \sqrt{49 \times 10^{-6}} = 7.0 \times 10^{-3}$$

Were you startled by the result in the second step? You shouldn't have been. When 10^{-5} is divided by 10, it becomes 10^{-6}, not 10^{-4}.

For practice, do this one: $\sqrt{6.4 \times 10^{-3}}$

(16)

A 0.08

B 0.8

YOUR ANSWER --- B

You are correct. After equalizing powers of ten and shifting the decimal points properly, your columnar arrangement looked like this:

$$\begin{array}{l} 0.0131 \times 10^4 \\ 8.45 \quad \times 10^4 \\ \underline{0.481 \quad \times 10^4} \end{array}$$

Yielding ----- 8.9441×10^4

Finally, since the least precise number (8.45×10^4) has only 2 decimal places, you rounded back the answer to 2 decimal places, writing it as $\underline{8.94 \times 10^4}$.

NOTEBOOK ENTRY

5. Addition and Subtraction in Scientific Notation

(a) Locate the largest exponent in the group to be added, then change all the remaining powers of ten to conform.

(b) Shift decimal points to compensate for each change in the power of ten.

(c) Add or subtract the coefficients in the usual manner to obtain the sum coefficient; retain the same exponent in the sum power of ten.

(d) Round back the answer to the same decimal place as the least precise measurement.

Using the rules in the NOTEBOOK ENTRY, subtract 9.38×10^3 from 2.25×10^4. Write your answer.

Please turn to page 36 to check your answer.

YOUR ANSWER --- C

You are correct. The coefficient, 5.2948, is larger than 5.0000. Thus, we multiply the power of ten by 10 and drop the coefficient. That is, $(10^{-10}) \times 10^{1} = 10^{-9}$, so the order of magnitude of the radius of a hydrogen electron's orbit is 10^{-9} m.

In a moment you will be looking at a table showing the order of magnitude of time duration of several things. You will notice that many of these have durations which are quite variable. For example, the human life span may range from 50 years to 100 years, depending on the individual. Therefore, to state the human life span as an exact number of years would be meaningless. Similarly, we cannot state exactly the time required to complete a baseball game because some of them last considerably longer than others. It is appropriate to state these times in terms of orders of magnitude.

As you study the table, you will probably be struck by the fact that it covers a very wide range of time intervals. The use of orders of magnitude for comparing vastly different measurements of any kind is also appropriate and meaningful.

Please go on to page 10.

TABLE 1

Orders of Magnitude of Times	
Event	Time Interval in SECONDS
Time elapsed since the first land life appeared on Earth.	10^{17}
Time elapsed since the dinosaurs became extinct.	10^{15}
Time elapsed since earliest man appeared on Earth.	10^{13}
Time elapsed since the death of Christ and the beginning of Christianity.	10^{11}
Human life span (please remember that all intervals are measured in seconds).	10^{9}
Time required for the Earth to revolve once around the Sun (year).	10^{7}
One month.	10^{6}
One day.	10^{5}
Duration of average baseball game in seconds.	10^{4}
One minute.	10^{2}
Time required to wind your wristwatch.	10^{1}
Time between heartbeats.	10^{0}
Time needed for the blades of an electric fan to turn just once.	10^{-2}
Time for a high-speed bullet to move a distance of a millimeter or two.	10^{-6}
Time for light to cross from one wall to the opposite wall in an average room.	10^{-8}
Time for light to pass through a window pane.	10^{-11}
Time for the hydrogen electron to revolve once about the nucleus in the atom.	10^{-15}
Time for the proton to spin once on its axis in the nucleus of an atom.	10^{-22}

(Adapted from P.S.S.C.)

Please turn to page 115 in the blue appendix.

Here again is the top half of the table:

TABLE 1

Orders of Magnitude of Times

Event	Time interval in SECONDS
Time elapsed since the first land life appeared on Earth.	10^{17}
Time elapsed since the dinosaurs became extinct.	10^{15}
Time elapsed since earliest man appeared on Earth.	10^{13}
Time elapsed since the death of Christ and the beginning of Christianity.	10^{11}
Human life span (please remember that all intervals are measured in seconds).	10^{9}
Time required for the Earth to revolve once around the Sun (year).	10^{7}
One month.	10^{6}
One day.	10^{5}

It is conventional to say, for example, that a year is two orders of magnitude greater than a day. How many orders of magnitude greater is the time since earliest man than the human life span?

(22)

A 10^4

B Four

YOUR ANSWER --- B

Your terms are confused. The exponent in this case is the power to which 10 is raised, or "-3." The portion "6.5" has an entirely different name.

Please return to page 3 and select another answer.

YOUR ANSWER --- C

Let's see what the error is. $10^3 = 1,000$, so that if we multiply $10^3 \times 10^3$, we are really multiplying $1,000 \times 1,000$. The product is then,

$$1,000 \times 1,000 = 1,000,000$$

To write this product, 1,000,000, as a power of ten we note that it is 10^6, not 10^9. The wrong answer above was obtained by multiplying the exponents. We have already shown you that this procedure is incorrect.

Please return to page 56 and select another answer.

YOUR ANSWER --- B

What happened to the decimal point? Each of the factors (5.6 and 2.1) has 1 decimal place; hence the product must have 2 decimal places. You were possibly a little careless.

Please return to page 95 and select the correct answer.

YOUR ANSWER --- B

You ignored the requirement for significant figures. This answer is not <u>wrong</u> in a purely arithmetic sense, but it does violate the significant figure product rule. How many significant figures are there in the least precise of these measurements: 5.00×10^4 m and 1.11×10^5 m? They have 3 significant figures each, don't they? So--how come you chose an answer with 5 significiant figures? Perhaps you forgot that the 2 zeros after the decimal .55 <u>are</u> significant.

Please return to page 96 and select another answer.

YOUR ANSWER --- B

This is incorrect. There is no error in this solution.

$1.86 \times 2.5 \times 10^5 \times 10^{-2} = 4.65 \times 10^3$ (remember the algebraic addition of the exponents). Then, finally:

$$4.65 \times 10^3 = 4.7 \times 10^3 \text{ (to 2 significant figures)}$$

Please return to page 42 and select another answer.

YOUR ANSWER --- A

There is an error in the first group. The first two items are correct, but the third one is wrong. Here it is corrected:

$$(8.00 \times 10^{-2})^2 = 64.0000 \times 10^{-4} = 6.4 \times 10^{-3}$$

The answer given was 6.4×10^{-4}. The mistake was this: When the decimal point was moved 1 place to the left to convert 64.000 to 6.4, the exponent was not changed to compensate for this division.

Please return to page 57 and select another answer.

YOUR ANSWER --- A

You performed one operation incorrectly. You're supposed to find the square root of the coefficient after having shifted the decimal point to obtain an even exponent. In other words,

$$\sqrt{8.100 \times 10^{11}} = \sqrt{81.00 \times 10^{10}}$$

Now the coefficient is 81.00. The square root of 81 is 9. Thus, you couldn't possibly get a coefficient of 40.50 if you did it properly. What you _did_ do is divide the original coefficient by 2 instead of finding its square root.

Please return to page 35 and select another answer.

CORRECT ANSWER:

(1) Setting up in scientific notation:

$$\frac{(6.20 \times 10^2)^2 \times (5.41 \times 10^{-4})}{(1.80 \times 10^2) \times (2.23 \times 10^{-4})}$$

(2) Squaring the first term in the numerator:

$$\frac{(3.844 \times 10^5) \times (5.41 \times 10^{-4})}{(1.80 \times 10^2) \times (2.23 \times 10^{-4})}$$

Note that the coefficient of the squared term is allowed to retain 4 significant figures; since this is an intermediate operation, the number should carry 1 significant figure more than we expect in the final answer.

(3) Collecting similar terms:

$$\frac{3.844 \times 5.41}{1.80 \times 2.23} \times \frac{10^5 \times 10^{-4}}{10^2 \times 10^{-4}}$$

(4) Performing the indicated operations yields the answer:

5.19×10^3 or 5,190

Here's one more for you. Take your time; do it right.

$$\frac{\sqrt{0.0144} \times (23,000)}{(520)^2 \times 2,500}$$

Please turn to page 47 to check your work.

YOUR ANSWER --- B

The arrangement is incorrect. The prime rule in adding columns of figures is that the unit column must form a straight vertical line, the tens column must form another straight vertical line, the hundreds column must do the same, and so forth. You can be sure that the columns are correctly aligned by writing the numbers so that decimal points, written or implied, are directly below each other in a straight vertical line.

Please return to page 48 and select another answer.

YOUR ANSWER --- A

You are correct. The solution follows:

Converting: $4.52 \times 10^2 = 0.452 \times 10^3$

Adding:
$$\begin{array}{r} 6.75 \times 10^3 \\ +\ 0.452 \times 10^3 \\ \hline 7.202 \times 10^3 \end{array}$$

Rounding back: $7.202 \times 10^3 = \underline{7.20 \times 10^3}$

The use of scientific notation makes it easy to determine what is called the order of magnitude of a measurement. The order of magnitude is the power of ten which is closest to the number. Giving the order of magnitude is a very approximate way of describing the number, but it often serves a useful purpose.

To illustrate, a rope is 132 meters long. We say its order of magnitude is 10^2 because 132 is closer to 100 than it is to 1,000. Writing it in scientific notation shows this clearly:

$$132 = 1.32 \times 10^2$$

Since 1.32 is less than 5.00, or less than 50% of the way to 10^3, we ignore it and say that the order of magnitude of 132 is 10^2 m.

Please go on to page 22.

Another example: A small sphere has a mass of 0.00368 kg. In scientific notation this is expressed as 3.68×10^{-3} kg. The coefficient is less than 5.00, so it is ignored in expressing the order of magnitude of the ball's mass. The order of magnitude of a 0.00368 kg mass is 10^{-3}.

One more illustration: The distance to the Sun from the Earth is about 93,000,000 miles. To find the order of magnitude, we write this number in scientific notation:

$$93{,}000{,}000 = 9.3 \times 10^{7}$$

Since the coefficient, 9.3, is greater than 5.00, we add one to the exponent, making it 10^{8}, and drop the 9.3 for purposes of order of magnitude. Thus, the order of magnitude of 93,000,000 miles is 10^{8} miles.

What is the order of magnitude of the distance of the Moon from the Earth if it is approximately 240,000 miles away?

(20)

A 10^{6} mi.

B 10^{5} mi.

C 10^{4} mi.

YOUR ANSWER --- B

You are correct. You're right on the ball! According to Table 1, the duration of an average game is of the order of 10^4 sec, while the time required for one rotation of the fan is of the order of 10^{-2} sec. The difference between exponents taken algebraically is: $4 - (-2) = 4 + 2 = \underline{6}$. Hence, the game is 6 orders of magnitude greater in duration than the time required for one rotation of the electric fan.

We have a few more questions based on TABLE 1.

Table 1 covers a certain range of orders of magnitude. A range is defined as the total count of numbers between two extreme values in an ordered listing without gaps, including both extremes. If a list has no gaps, you can obtain the range by counting. For example, look at this series:

3, 4, 5, 6, 7, 8

The range of this sequence, including both extremes, is 6. (Count the figures.) You can also obtain the range by algebraically subtracting the lowest extreme from the highest and adding 1: $8 - 3 + 1 = 6$

Now look at the same series with a gap in it (the 5 is missing):

3, 4, 6, 7, 8

When there are gaps in a series, we cannot obtain the range by counting; we must instead use the second method outline above. The range of this series is 6:

$$8 - 3 + 1 = 6$$

Please go on to page 24.

TABLE 1

Orders of Magnitude of Times

Event	Time Interval in SECONDS
Time elapsed since the first land life appeared on Earth.	10^{17}
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Time elapsed since earliest man appeared on Earth.	10^{13}
Time elapsed since the death of Christ and the beginning of Christianity.	10^{11}
Human life span (please remember that all intervals are measured in seconds).	10^{9}
Time required for the Earth to revolve once around the Sun (year).	10^{7}
One month.	10^{6}
One day.	10^{5}
Duration of average baseball game in seconds.	10^{4}
One minute.	10^{2}
Time required to wind your wristwatch.	10^{1}
Time between heartbeats.	10^{0}
Time needed for the blades of an electric fan to turn just once.	10^{-2}
Time for a high-speed bullet to move a distance of a millimeter or two.	10^{-6}
Time for light to cross from one wall to the opposite wall in an average room.	10^{-8}
Time for light to pass through a window pane.	10^{-11}
Time for the hydrogen electron to revolve once about the nucleus in the atom.	10^{-15}
Time for the proton to spin once on its axis in the nucleus of an atom.	10^{-22}

(Adapted from P.S.S.C.)

What would you say is the total range of orders of magnitude presented in TABLE 1?

(24)

A The range of the table is 22 orders of magnitude.

B The range of the table is 39 orders of magnitude.

C The range of the table is 40 orders of magnitude.

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

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CORRECT ANSWERS:

(a) $\sqrt{225} = 15$ (b) $\sqrt{900} = 30$ (c) $\sqrt{256} = 16$

Taking the square root of a number is really a special form of division. Perhaps it can best be illustrated by working an example in reverse.

Suppose you square the number 3×10^4. This is what you have according to the rule for squaring numbers in scientific notation:

$$(3 \times 10^4)^2 = 9 \times 10^8$$

(See Notebook Entry, Lesson 3, item 2.) Now let us find the square root of 9×10^8.

$\sqrt{9 \times 10^8} = 3 \times 10^4$. This is the opposite process to squaring a number.

How was the square root of this number in scientific notation obtained? First, you found the square root of the coefficient, 9. Second, you divided the exponent by 2. So you see that finding a square root is exactly the reverse of squaring a number in scientific notation.

Try one for yourself.

What is the square root of 9×10^{16}?

(14)

A 3×10^8

B 3×10^4

CORRECT ANSWER: 4.52×10^{-2} or 0.0452

It is often helpful to collect similar terms in a problem such as this one.

$$\frac{3.56 \times 3.45}{2.72} \times \frac{10^4 \times 10^{-3}}{10^3}$$

You might then operate on the coefficients as indicated. This gives an answer of 4.52 for the coefficient portion.

Then you might take care of the exponential portion. Thus, $10^4 \times 10^{-3} = 10^1$; then 10^1 divided by $10^3 = 10^{-2}$.

The entire answer is, then

$$4.52 \times 10^{-2} \quad \text{or} \quad 0.0452.$$

Set this one up in scientific notation and work it out.

$$\frac{(620)^2 \times 0.000541}{180 \times 0.000223}$$

Please turn to page 19 to check your work.

YOUR ANSWER --- A

Let's see why this is incorrectly done. $10^2 = 100$ and $10^4 = 10{,}000$. Multiplying these the long way, we have:

$$100 \times 10{,}000 = 1{,}000{,}000$$

Writing 1,000,000 as a power of ten, we see that it is 10^6. Thus, $10^2 \times 10^4 = 10^6$, not 10^8. The wrong answer above was obtained by multiplying the exponents. We have already shown you that this procedure is incorrect.

Please return to page 56 and select another answer.

YOUR ANSWER --- D

This is incorrect. In squaring a number written in scientific notation, you must square the coefficient and double the exponent. You <u>squared</u> <u>both</u> the coefficient and the exponent. This does not follow the correct rules of arithmetic.

Please return to page 99 and select another answer.

YOUR ANSWER --- A

Is that what was done in the example?

$$\frac{7.5 \times 10^5}{3 \times 10^2} = 2.5 \times 10^3$$

If the double subtraction you suggest is performed, we end up with:

$$\frac{7.5 \times 10^5}{3 \times 10^2} = 4.5 \times 10^3$$

And this is the wrong answer. Look over the example again. How can you get 2.5 by using 7.5 and 3? Certainly not by subtracting one from the other! How, then?

Please return to page 100 and make a more logical choice.

YOUR ANSWER --- B

Your selection was wrong. This solution is perfectly correct. Here--check it over step by step.

(1) 6.3 divided by -9.0 is an example of a positive number divided by a negative number to yield a negative quotient. Thus, the quotient is -0.7. We will later change this to -7.0 to convert it to the preferred form for scientific notation.

(2) 10^{14} divided by 10^{-6} is handled by subtracting (-6) from 14. To do this, we change the sign of the -6 to +6, then add algebraically to obtain 10^{20}.

(3) Thus far we have: -0.7×10^{20}.

(4) Now, changing to scientific notation, we multiply the -0.7 by 10, getting -7.0. To compensate, we divide the power of ten by 10, going from 10^{20} to 10^{19}. This finally gives us:

$$-7.0 \times 10^{19}$$

So you see that the example was worked out correctly.

Please return to page 87 and go on with the solutions.

YOUR ANSWER --- A

Remember that failure to keep a proper notebook will bounce back at you. The statement you selected is not item 1(c). It is an incomplete statement of another item in the same group.

Please return to page 88 and select another answer.

YOUR ANSWER --- A

You are correct. You followed the rule properly; you found the square root of 9 to be 3 and you then divided 16 by 2 to determine the new exponent.

NOTEBOOK ENTRY

4. Square Root in Scientific Notation

(a) Inspect the number to be sure that the exponent of 10 is even. i. e., divisible by 2. If it is even, proceed to step (c) below.

(b) If the exponent of 10 is odd, shift the decimal point in the coefficient one place to the right and compensate for this multiplication by dividing the power of ten by 10.

(c) Find the square root of the coefficient and divide the exponent by 2. Combine the two in the usual manner, restoring the resulting number to proper scientific notation and the proper number of significant figures.

Please go on to page 35.

We shall clarify notebook entries (a) and (b) with a few examples.

Suppose we want to find the square root of 3.6×10^7. In accordance with rule 4(a), we inspect the number and find the exponent to be <u>odd</u>. So, we shift the decimal point one place to the right and divide the power of ten by 10. Thus:

$$\sqrt{3.6 \times 10^7} = \sqrt{36 \times 10^6}$$

Now we can follow rule 4(c). The square root of the coefficient is $\sqrt{36} = 6.0$. The exponent divided by 2 is $6/2 = 3$. Hence, our answer is <u>6.0×10^3</u>.

Here's one for you. Perform the indicated operation:

$$\sqrt{8.100 \times 10^{11}} = ???$$

(15)

A 40.50×10^5

B 9×10^5

C 9.000×10^6

D 9.000×10^5

CORRECT ANSWER: $(2.25 \times 10^4) - (9.38 \times 10^3) = \underline{1.31 \times 10^4}$

<u>Procedure</u>: (1) Convert 9.38×10^3 to 0.938×10^4.

(2) Columnize

$$\begin{array}{r} 2.25 \times 10^4 \\ \underline{-\ 0.938 \times 10^4} \end{array}$$

Difference -------- 1.312×10^4

(3) Round back to $\underline{1.31 \times 10^4}$.

Before continuing, please turn to page 114 in the blue appendix.

In the examples below, only <u>one</u> is completely correct. Select the correct example, and choose the corresponding letter. (Answers must be correct with respect to significant figures, too.)

(19)

A $(6.75 \times 10^3) + (4.52 \times 10^2) = 7.20 \times 10^3$

B $(1.88 \times 10^3) + (6.29 \times 10^5) = 6.31 \times 10^3$

C $(5.87 \times 10^8) - (8.36 \times 10^7) = -2.49 \times 10^8$

D $(1.38 \times 10^2) - (6.43 \times 10^{-2}) = 5.05 \times 10^{-1}$

YOUR ANSWER --- A

When you compare orders of magnitude, please don't do it in terms of powers of ten. For instance, one month is one order of magnitude greater than one day, not 10^1 orders of magnitude greater. When we say that a year is two orders of magnitude greater than a day, we mean that a year is 10^2 times as long as a day.

Although it is true that the time since earliest man appeared on earth is 10^4 times as long as the time of one human life span, you cannot say that one is 10^4 orders of magnitude greater than the other.

Please return to page 11 and select another answer.

YOUR ANSWER --- A

Not correct. Let's look at the figures. According to Table 1, the duration of the average baseball game is of the order of 10^4 sec, while the time required for one rotation of the electric fan is of the order of 10^{-2} sec.

To determine the number of orders of magnitude greater the game time is compared to the fan time, you should subtract the smaller exponent from the larger algebraically.

Thus, 4 - (-2) will give you the answer.

You apparently ignored the minus sign before the 2. This would give you the wrong answer, of course.

Please return to page 64 and select another answer.

Your work on this one should have looked like this:

$$25 - (-15) + 1 = 41$$

This is the correct answer.

Please go on now to page 66.

YOUR ANSWER --- A

You can't. When you multiply 3 x 2, you get 6. The exponent in our answer is 5. So, in multiplying powers of ten, you do not multiply the exponents of the multiplicand and multiplier to get the exponent of the product.

Please return to page 68 and select another answer.

YOUR ANSWER --- A

You are correct. The coefficients are multiplied, as any two numbers are multiplied, pointing off decimal places in the usual manner.

However, now that we're writing scientific notation, we don't like to see the decimal point after the second digit in the coefficient of the product. So, let's rewrite it properly:

$$11.76 \times 10^5 = 1.176 \times 10^6$$

We moved the decimal point one place to the left to place it after the first digit. This is the equivalent of dividing the coefficient by 10, hence we multiplied the power of ten by 10, changing it from 10^5 to 10^6.

Now, suppose the original numbers were measurements, say, of weights. They might have been 5.6×10^3 kg and 2.1×10^2 kg. Both of these are expressed to 2-significant figure precision. Therefore, how many significant figures should the product of these two weights contain?

(5)

A 2 significant figures.

B 4 significant figures.

YOUR ANSWER --- A

You are correct. Here is the example worked out:

$$\begin{aligned}(5.00 \times 10^4 \text{ m}) \times (1.11 \times 10^5 \text{ m}) &= 5.00 \text{ m} \times 1.11 \text{ m} \times 10^9 \\ &= 5.5500 \times 10^9 \text{ m}^2 \\ &= 5.55 \times 10^9 \text{ m}^2\end{aligned}$$

Now, we want to find the area of a long, narrow rectangle measuring 0.520 cm by 123.5 cm. In scientific notation we have:

$$(5.2 \times 10^{-1} \text{ cm}) \times (1.235 \times 10^2 \text{ cm})$$

We see that we have one positive and one negative exponent. How are these handled? Addition of exponents is <u>algebraic</u>, so that we must take the <u>sign</u> into account. Thus:

$$\begin{aligned}&5.2 \text{ cm} \times 1.235 \text{ cm} \times 10^1 \\ &= 6.4220 \times 10^1 \text{ cm}^2 \\ &= 6.4 \times 10^1 = 64 \text{ cm}^2\end{aligned}$$

So, there is no change in our rules so long as we remember to handle the signs of the exponents algebraically.

<u>One</u> of the solutions below contains an error. Which one is it? For simplicity, we shall omit units but want you to solve these as measurements, giving attention to significant figures.

(7)

A $(6.100 \times 10^{-3}) \times (9.2 \times 10^8) = 5.6 \times 10^5$

B $(1.86 \times 10^5) \times (2.5 \times 10^{-2}) = 4.7 \times 10^3$

C $(8.03 \times 10^{-12}) \times (6.12 \times 10^{-6}) = 4.91 \times 10^{-17}$

YOUR ANSWER --- A

This is incorrect. In squaring a number written in scientific notation, you must square the coefficient and double the exponent. Your answer was obtained by doubling both coefficient and exponent. In other words, the square of 2.5 is not 5.00.

Please return to page 99 and select another answer.

YOUR ANSWER --- C

You are correct. Good work!

Before continuing, please turn to page 112 in the blue appendix.

To keep your notebook complete, let's phrase the rule carefully for squaring numbers in scientific notation, and make the necessary entry.

NOTEBOOK ENTRY

2. Squaring Numbers in Scientific Notation

(a) To square a number, square the coefficient and double the exponent, following the rules for handling algebraic signs throughout the operation. (The square of any quantity, whether positive or negative, is positive; when a positive number is doubled, the product is positive; when a negative number is doubled, the product is negative.)

(b) The square of a number in scientific notation should be restored to proper scientific notation form by moving the decimal point to place it after the first non-zero digit and appropriately compensating for this movement by changing the exponent (with the same exception noted in rule 1).

Please go on to page 45.

We're prepared now for <u>division</u> in scientific notation. You are aware, of course, that the process of division is just the reverse of multiplication. You would expect, then, that the actual mechanics of division would follow an opposite pattern. This is quite true, as the following example illustrates.

Divide 100,000 by 100.

$$\frac{100{,}000}{100} = 1{,}000 \text{ or } \frac{10^5}{10^2} = 10^3$$

Thus, in dividing one power of ten by another, what do we do with the exponents?

(10)

A Divide the exponent of the numerator by that of the denominator.

B Subtract the exponent of the denominator from that of the numerator.

YOUR ANSWER --- C

You performed one of the operations incorrectly. When the decimal point is moved to the right to obtain an even exponent, you're supposed to divide the exponent by 10. You multiplied it by 10 instead. This is what you did:

This should be 10, not 12.

$$\sqrt{8.100 \times 10^{11}} = \sqrt{81.00 \times 10^{12}} = 9.000 \times 10^{6}$$

You deserve congratulations anyway, because you did get the right number of significant figures into your answer, and you did know how to find the square root of 81.00. But, be careful about the exponent value when you move the decimal point.

Please return to page 35 and select another answer.

THE CORRECT ANSWER:

(1) Set up in scientific notation:

$$\frac{\sqrt{1.44 \times 10^{-2}} \times (2.30 \times 10^{4})}{(5.20 \times 10^{2})^{2} \times \sqrt{2.50 \times 10^{3}}}$$

(2) Square and find square roots as indicated:

$$\frac{(1.20 \times 10^{-1}) \times (2.30 \times 10^{4})}{(2.704 \times 10^{5}) \times (5.00 \times 10^{1})}$$

(3) Collect similar terms:

$$\frac{1.20 \times 2.30}{2.704 \times 5.00} \times \frac{10^{-1} \times 10^{4}}{10^{5} \times 10^{1}}$$

(4) Perform the indicated operations and obtain the answer:

$$2.04 \times 10^{-4}$$

We shall end this portion of our work with a discussion of addition and subtraction using scientific notation. It turns out that these operations can be performed much more easily than multiplication, division, or working with squares and roots.

Please turn to page 113 in the blue appendix.

A basic fact that you have been using since you were in grade school is that, when adding or subtracting decimal numbers, you always columnize the figures so that the decimal points are directly under one another.

To add the figures 27.842, 3740, and 0.066, which arrangement below is correct?

(17)

```
     27.842
A  3740.
       .066

   27.842
B  3740.
  .066

   27.842
C    3740
     .066
```

YOUR ANSWER --- B

The power of ten is wrong. Let's run through it.

Converting: 1.88×10^3 to the higher exponent:
$1.88 \times 10^3 = 0.0188 \times 10^5$

Adding:
$$\begin{array}{r} 0.0188 \times 10^5 \\ \underline{6.29 \times 10^5} \\ 6.3088 \times 10^5 \end{array}$$

Rounding back: $6.3088 \times 10^5 = 6.31 \times 10^5$

You see that the coefficient was obtained correctly, but the power of ten was wrong.

Please return to page 36 and select another answer.

YOUR ANSWER --- A

This is not correct. Expressing 240,000 miles in scientific notation, we have:

$$240,000 \text{ mi} = 2.4 \times 10^5 \text{ mi}$$

Now ask yourself whether or not the coefficient, 2.4, is closer to 1 or to 10. In other words, does the coefficient "carry" the number more or less than 50% toward the next higher power of ten. The answer is that it is less than 50% of the way toward the next higher power of ten.

You should now have all the clues you need.

Please return to page 22 and choose another answer.

YOUR ANSWER --- A

Perhaps you're not sure of the meaning of the word "range." The total count of numbers between two extreme values in an ordered listing without gaps and including both extremes, is the range of the listing.

Let's take a simple example. Look at this list:

3 4 5 6 7 8	The range is six; that is, there are 6 numbers listed with no gaps between them. You can determine the range either by counting the numbers or by subtracting the smaller from the larger and adding 1. Thus, $8 - 3 + 1 = 6$

Now consider this list:

3 2 1 0 -1 -2 -3	The range is seven in this case. Count the numbers; there are seven of them with no gaps between them. Now subtract algebraically and add 1. $3 - (-3) + 1 = 7$.

Thus, the range may be obtained by subtracting the smallest limit from the largest and adding 1, whether or not there is a zero included in the list.

In selecting your answer, you merely assigned a range to the table by using the largest exponent you saw and changing its sign. This won't work.

Please return to page 24 and select another answer.

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YOUR ANSWER --- C

You are correct. If you selected this on the first try, it probably means that you are exercising good care in note taking.

NOTEBOOK ENTRY

(topic 3)

(e) Examples of Division

$$\frac{-4.8 \times 10^{-7}}{6.0 \times 10^{4}} = -8.0 \times 10^{-12}$$

$$\frac{6.3 \times 10^{14}}{-9.0 \times 10^{-6}} = -7.0 \times 10^{19}$$

$$\frac{-9.6 \times 10^{-6}}{-1.2 \times 10^{-9}} = 8.0 \times 10^{3}$$

Many equations in physics contain factors under a radical; that is, it is often necessary to take the square root of numbers expressed in scientific notation. The square root of a number, as you know, is that number which, when multiplied by itself, gives a product equal to the original number. Thus, the square root of 9 is 3; the square root of 16 is 4; the square root of 25 is 5. This operation may be indicated by the radical $\sqrt{36} = 6$.

Just as a brush up, solve the following.

(a) $\sqrt{225}$ = ??? (b) $\sqrt{900}$ = ??? (c) $\sqrt{256}$ = ???

Please turn to page 27 to check your answers.

YOUR ANSWER --- B

You are correct. To multiply $10^3 \times 10^2$, you merely <u>add</u> the exponents to obtain 10^5. Similarly, to multiply 10,000 x 1,000, you would write this as $10^4 \times 10^3$, add the exponents, and note the product as 10^7. You will notice that this process of adding exponents is really the same thing as adding the zeros of one of the numbers to the other, just as we concluded previously.

In the three examples below, only <u>one</u> of the operations has been handled properly. Which one is it?

(3)

A $10^2 \times 10^4 = 10^8$

B $10^5 \times 10^3 = 10^8$

C $10^3 \times 10^3 = 10^9$

YOUR ANSWER --- C

You are correct. Good work. You squared the coefficient and doubled the exponent according to the rules. That is:

$$(25{,}000)^2 = (2.5 \times 10^4)^2 = 6.25 \times 10^8 \text{ mi}^2$$

You often encounter numbers in which the exponent is negative. Since 2 times a negative quantity yields a negative product, then the exponent when doubled would still be negative. For example,

$$4.3 \times 10^{-3} = 18.49 \times 10^{-6} = 1.8 \times 10^{-5} \text{ (2 sig. figs.)}$$

In the practice groups below, only one of the groups is <u>wholly</u> correct. <u>Do them all</u>; then select the right answer.

(9)

A This group is correct.

$(1.33 \times 10^2) \times (6.42 \times 10^3) = 8.54 \times 10^5$
$(7.10 \times 10^4) \times (6.42 \times 10^{-8}) = 4.56 \times 10^{-3}$
$(8.00 \times 10^{-2})^2 = 6.4 \times 10^{-4}$

B This group is correct.

$(7.32 \times 10^{16}) \times (2.55 \times 10^2) = 1.87 \times 10^{18}$
$(2.43 \times 10^{-3}) \times (3.05 \times 10^{-4}) = 7.41 \times 10^{-7}$
$(4.21 \times 10^4)^2 = 1.77 \times 10^9$

C This group is correct.

$(9.20 \times 10^{-3}) \times (3.05 \times 10^{-4}) = 2.81 \times 10^{-6}$
$(-3.04 \times 10^{-2})^2 = 9.24 \times 10^{-4}$
$(6.82 \times 10^{20})^2 = 4.65 \times 10^{41}$

YOUR ANSWER --- C

Was this done in the example? Properly solved it is:

$$\frac{7.5 \times 10^5}{3 \times 10^2} = 2.5 \times 10^3$$

If the double division you suggest is performed, we end up with:

$$\frac{7.5 \times 10^5}{3 \times 10^2} = 2.5 \times 10^{2.5}$$

And this is the wrong answer.

Look over the example again. How can you obtain 10^3 by operating on 10^5 and 10^2? Certainly not by dividing one into the other!

How then?

Please return to page 100 and select another answer.

YOUR ANSWER --- C

You are correct. This solution is wrong. We'll check it over step by step to locate the error.

(1) -9.6 divided by -1.2 is an example of a negative number divided by a negative number to yield a positive quotient. Hence, the quotient is 8.0.

(2) 10^{-6} divided by 10^{-9} is handled by subtracting -9 from -6. To do this, we change the sign of the -9 to +9, and then add algebraically. This gives us 10^3.

(3) Putting coefficient and power of ten together we have:

$$8.0 \times 10^3$$

The error is now clear. The original answer shows an exponent of -3 where it should be +3.

NOTEBOOK ENTRY

3. Division in Scientific Notation

(a) Divide the coefficients algebraically, giving attention to proper algebraic sign.

(b) Divide the powers of ten by algebraically subtracting the exponent in the denominator from the exponent in the numerator, again giving attention to algebraic sign rules.

(c) Round back the coefficient to the proper number of significant figures.

(d) Combine the quotient coefficient with the quotient power of ten by means of a "x" sign, and move the decimal point to conform with accepted scientific notation.

Please go on to page 60.

With the rules for division stored where they can be used, we can now fulfill a promise made some time ago. In an earlier lesson, we asked you to accept on faith the fact that any number raised to the zero power is equal to 1. We can now prove that this is so, using a reverse approach.

We first recognize that any number divided by itself equals 1. That is:

$$\frac{51}{51} = 1 \qquad \frac{-18}{-18} = 1 \qquad \frac{6.543}{6.543} = 1 \quad \text{etc.}$$

Any number can be expressed in terms of some base raised to the proper power. For example, $64 = 8^2$. Here, we are expressing the number 64 in terms of the base 8 raised to the 2nd power. Or, $27 = 3^3$, where the base is 3 raised to the 3rd power. Of course, we're selecting easy numbers as illustrations, but the same rule applies to any number. Since we are particularly interested in convincing you that $10^0 = 1$, let us work with powers of ten.

Please go on to page 61.

Suppose we choose a number at random, for example, 7391. Now let us express this in scientific notation:

$$7.391 \times 10^3$$

As our next step, we will divide this number by itself, obtaining 1 as an answer.

$$\frac{7.391 \times 10^3}{7.391 \times 10^3} = 1.000 \times 10^0$$

We arrived at 10^0 here by applying the rules for division of powers of ten. Now, since we know that any number divided by itself equals 1, then we know that:

$$1.000 \times 10^0 = 1$$

Thus, the only value 10^0 can possibly have is 1.

Let's see how this works out in another example. The number 1,086 is exactly twice 543; hence if we divide 1,086 by 543, we must get an answer of 2. Expressing both in scientific notation, we can write:

$$\frac{1.086 \times 10^3}{5.43 \times 10^2}$$

Dividing, we obtain

$$0.200 \times 10^1$$

Shifting the decimal point one place to the right to convert the coefficient to proper form, we have:

$$2.00 \times 10^0$$

Since we knew in advance that the answer must be 2, then 10^0 must be equal to 1.

Please go on to page 62.

Work out all the examples below in writing, then check your answers as directed. <u>Use scientific notation throughout</u>. All the numbers are correct to 3 significant figures.

(1) 753 divided by 24.0
(2) 24,300 divided by 0.0366
(3) 0.000622 divided by 0.00622
(4) 2,730 divided by 2,700
(5) 0.533 divided by 0.401

Please go on to page 88 for the correct answers.

YOUR ANSWER --- B

Right. In scientific notation, 240,000 = 2.4×10^5. The coefficient, 2.4, is less than 5.0. This means that the entire number is closer to 10^5 than it is to 10^6. So--the order of magnitude of 240,000 is 10^5.

Very accurate laboratory measurements in the past few decades have revealed that the mass of an electron is 9.1083×10^{-31} kg. As you see, 9.1083 is substantially larger than 5.0000 so that the order of magnitude of the mass of an electron is 10^{-30} kg. Be careful with negative exponents. Always be aware that reducing the numerical value of a negative exponent makes the power of ten larger, as in this example. That is, since 9.1083 is larger than 5.000, we must increase 10^{-31} to the next next higher order of magnitude. To do this, we change 10^{-31} to 10^{-30}.

If you understand this, you'll have no trouble with this one: The radius of the electron orbit around the nucleus in a normal hydrogen atom is 5.2948×10^{-10} meters. What is the order of magnitude of the orbit?

(21)

A 10^{-11} m

B 10^{-10} m

C 10^{-9} m

YOUR ANSWER --- B

You are correct. To find the orders of magnitude between any two measurements, that is, to find out how many orders of magnitude greater one is as compared to the other, you merely subtract one exponent from the other algebraically. In this case, the original orders of magnitude were 10^{13} for earliest man and 10^9 for human life span. Subtracting exponents, we may write $13 - 9 = 4$. This is the number of orders of magnitude between the two measurements.

Remember that the subtraction must be algebraic. The usual sign conventions are to be followed.

O. K. Try this one: the duration of a baseball game is about 10^4 sec, according to Table 1. The time required for an electric fan to complete one rotation is about 10^{-2} second. How many orders of magnitude greater is the duration of the ball game than the time needed for one rotation of the fan?

(23)

A Two

B Six

C 10^2

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You have now completed the study portion of Lesson 3 and your Study Guide Computer Card and A V Computer Card should be properly punched in accordance with your performance in this Lesson.

You should now proceed to complete your homework reading and problem assignment. The problem solutions must be clearly written out on $8\frac{1}{2}$" x 11" ruled, white paper, and then submitted with your name, date, and identification number. Your instructor will grade your problem work in terms of an objective preselected scale on a Problem Evaluation Computer Card and add this result to your computer profile.

You are eligible for the Post Test for this Lesson only after your homework problem solutions have been submitted. You may then request the Post Test which is to be answered on a Post Test Computer Card.

Upon completion of the Post Test, you may prepare for the next Lesson by requesting the appropriate:

1. study guide
2. program Control Matrix
3. set of computer cards for the lesson
4. audio tape

If films or other visual aids are needed for this lesson, you will be so informed when you reach the point where they are required. Requisition these aids as you reach them.

Good Luck!

YOUR ANSWER --- C

You are correct. In this example, "10^{-3}" is called the power of ten; the "-3" is called the exponent; and the "6.5" is called the coefficient.

Please go on to page 68.

We'll start with multiplication of numbers expressed in scientific notation. First, we'll multiply 1,000 by 100.

$$\begin{array}{r} 1{,}000 \\ \underline{\times \quad 100} \\ 100{,}000 \end{array}$$

Obviously, to multiply two simple powers of ten, you merely add the zeros of one of the numbers to the other number, just as we did in the example. Similarly, 10,000 x 1,000 = 10,000,000. Here we have tacked the 3 zeros that belong to the "1000" onto the "10,000" to obtain an answer of 10,000,000.

Let us rewrite the example above in this form:

$$\begin{array}{rcr} 1{,}000 & \text{---------} & 10^3 \\ \underline{\times \quad 100} & \text{----------} & \underline{\times \ 10^2} \\ 100{,}000 & & 100{,}000 = 10^5 \end{array}$$

Note what has happened. We wrote 1,000 as 10^3 and 100 as 10^2. Then, $10^3 \times 10^2$ must also yield an answer of 100,000. Now we express 100,000 as 10^5 and compare all the exponents. How could we have arrived at 10^5 from the product of 10^3 and 10^2 directly?

(2)

A By multiplying the exponents.

B By adding the exponents.

YOUR ANSWER --- B

You have forgotten the rule for multiplication of significant figures. A product should contain no more significant figures than the least precise measurement. The least precise measurement contains 2 significant figures. (In this case, both measurements contain the same number of significant figures.) Hence, the product cannot have 4 significant figures.

Please return to page 41 and select the correct answer.

YOUR ANSWER --- C

You have everything right <u>except</u> the preferred position of the decimal point in scientific notation. In your answer, the decimal point is understood after the third "5." In scientific notation, we want the decimal point after the first non-zero digit.

Please return to page 96 and select another answer.

YOUR ANSWER --- C

This is incorrect. There is no error in this solution.

$8.03 \times 6.12 \times 10^{-12} \times 10^{-6} = 49.1436 \times 10^{-18}$ (since the exponents add algebraically). Then, finally:

$$49.1436 \times 10^{-18} = 4.91436 \times 10^{-17} =$$
$$4.91 \times 10^{-17} \text{ (to 3 significant figures)}$$

Please return to page 42 and select another answer.

YOUR ANSWER --- B

Check again. In squaring a number written in scientific notation, you must square the coefficient and double the exponent. You did exactly the reverse of this: you doubled the coefficient and squared the exponent.

Please return to page 99 and select another answer.

YOUR ANSWER --- B

You were not sufficiently careful with the exponent.

Since 10^{-3} carries an odd exponent, you shifted the decimal point in the coefficient one place to the right so that you could divide the power of ten by 10 to make it even. This gives you

$$\sqrt{64 \times \frac{10^{-3}}{10^{1}}}$$

However, 10^{-3} divided by 10^{1} is 10^{-4}. Remember, when you divide a power of ten having a negative exponent by 10, the numerical value of the exponent increases by one.

The answer you selected indicates that you obtained 10^{-2} rather than 10^{-4} when you divided 10^{-3} by 10.

Please return to page 7 and select the correct answer.

YOUR ANSWER --- A

You are correct. The decimal points must form a straight vertical line if the digit columns are to be correctly aligned.

In scientific notation, as you know, the position of the decimal point is dictated by the exponent of the power of ten. As illustrations, consider these:

5×10^1 is really 50.
5×10^2 is really 500.
5×10^3 is really 5000.

In order to line up the decimal points vertically, we must change the numbers so that they all have the same exponent in their power of ten section. We can check this idea easily by means of the group of numbers above. Let's add them in standard notation first.

$$
\begin{array}{r}
50. \\
500. \\
\underline{5000.} \\
5550
\end{array}
$$

This sum may be written as 5.55×10^3.

Now, let's convert the first two (5×10^1 and 5×10^2) into a form such that they all have the same powers of ten, namely 10^3.

0.05×10^3 (This is 50 in this notation.)
0.5×10^3 (This is 500 in this notation.)
$5. \times 10^3$ (This is 5,000 in this notation.)

Please go on to page 75.

So, we arrive at the same answer by changing all the powers of ten to the same value, and then adding the coefficients only, while retaining the common power of ten. It is generally most convenient to use the largest power of ten of the group as the common value; in our example, the largest power of ten is 10^3, so all the others were converted to this.

Suppose you try adding the following group, giving attention to significant figures.

$$\begin{array}{l} 1.31 \times 10^2 \\ 8.45 \times 10^4 \\ \underline{4.81 \times 10^3} \end{array}$$

Which of these answers did you come out with?

(18)

A 14.57×10^4

B 8.94×10^4

YOUR ANSWER --- C

This is not correctly done. We'll work it out.

Converting: $8.36 \times 10^7 = 0.836 \times 10^8$

Subtracting:
$$\begin{array}{r} 5.87 \times 10^8 \\ -\ 0.836 \times 10^8 \\ \hline 5.034 \times 10^8 \end{array}$$

Rounding back: $5.034 \times 10^8 = 5.03 \times 10^8$

In obtaining the answer you chose, you forgot to shift the decimal point in the coefficient. This yielded a negative coefficient.

Please return to page 36 and select another answer.

YOUR ANSWER --- C

This answer is incorrect. You don't find the order of magnitude by expressing a power of ten for which the exponent is equal to the number of zeros. This would give us 24×10^4. If we took the distance to 3 significant figures we would get 239,000 miles and you might have written 239×10^3. Then your order of magnitude would be 10^3, if you used a rule like the one indicated. Using scientific notation,

$$240{,}000 \text{ mi} = 2.4 \times 10^5 \text{ mi}$$

Does the coefficient, 2.4 in this case, carry the number more than half the way up to the next power of ten? If it doesn't, then the order of magnitude is 10^5. If it does, then the order of magnitude is 10^6. Which one is it?

Please return to page 22 and select another answer.

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YOUR ANSWER --- D

This statement is <u>not</u> item 1(c) in your notebook. You need to be more impressed with the need for care in note-taking. The item you chose is a correct statement of item 1(a), not 1(c).

Please return to page 88 and select another answer.

CORRECT ANSWER:

$$\frac{(3.56 \times 10^4) \times (3.45 \times 10^{-3})}{2.72 \times 10^3}$$

You can approach this problem in one of several ways. You might multiply the factors in the numerator, then divide by the factor in the denominator. You might divide either factor in the numerator by the factor in the denominator and then multiply this quotient by the remaining numerator factor. There is absolutely no difference between these methods with regard to the final answer. So, choose your own weapon! Solve it any way you prefer, but solve it correctly!

Please turn to page 28 to check your answer.

YOUR ANSWER --- B

There is an error in Group 2. The first item is wrong, while the second and third are correct. The right way to do the first operation is this:

$$(7.32 \times 10^{16}) \times (2.55 \times 10^{2}) = 18.7 \times 10^{18}$$
$$= 1.87 \times 10^{19}$$

The answer given was 1.87×10^{18}. The mistake was this: when the decimal point was moved 1 place to the left to convert 18.7 to 1.87, the exponent was not changed to compensate for this division.

Please return to page 57 and select another answer.

YOUR ANSWER --- A

No. Look at the example again.

$$\frac{100,000}{100} = 1,000$$

There is no doubt about this, is there? Right. Now let's convert 100,000 and 100 into their respective powers of 10; that is, 10^5 and 10^2. Dividing:

$$\frac{10^5}{10^2} \text{ must also be equal to } 1,000$$

But 1,000 is 10^3. Hence, we can write this as:

$$\frac{10^5}{10^2} = 10^3$$

Then, it appears evident that we can obtain 10^3 only by subtracting the exponent of the denominator from the exponent of the numerator.

Perhaps a second example will help. Divide 1,000,000 by 10,000. In standard notation this yields:

$$\frac{1,000,000}{10,000} = 100$$

Now, expressing the same problem in powers of ten, we have:

$$\frac{10^6}{10^4} = 100, \text{ but } 100 = 10^2; \text{ hence } \frac{10^6}{10^4} = 10^2.$$

The answer, 10^2, can be obtained only by <u>subtraction</u> of the exponent of the denominator from the exponent of the numerator.

Please return to page 45 and select the correct answer.

YOUR ANSWER --- B

You are correct. In the example

$$\frac{7.5 \times 10^5}{3 \times 10^2} = 2.5 \times 10^3$$

the coefficient "2.5" is obtained by <u>dividing</u> 7.5 by 3, while the factor "10^3" is obtained by <u>subtracting</u> the exponent of 10^2 from that of 10^5.

As in multiplication and squaring, we often must deal with negative exponents. By now, you should have no trouble with the techniques since all you have to do is remember to apply the algebraic sign rules:

(1) A negative number divided by a negative number yields a positive quotient; a positive number divided by a positive number yields a positive quotient; and a division involving both a negative and a positive number yields a negative quotient.

(2) When subtracting, change the sign of the subtrahend, then add algebraically. The result carries the sign of the larger of the two quantities.

Now for a little practice. Write your answers to the following on some scrap paper.

(1) $\frac{6 \times 10^8}{2 \times 10^4} = ???$ (2) $\frac{6 \times 10^8}{2 \times 10^{-4}} = ???$

(3) $\frac{8 \times 10^{-5}}{2 \times 10^{-8}} = ???$ (4) $\frac{8 \times 10^{-5}}{2 \times 10^8} = ???$

Please go on to page 87.

THE CORRECT SOLUTIONS ARE:

(1) $\frac{6 \times 10^{8}}{2 \times 10^{4}} = 3 \times 10^{4}$ (2) $\frac{6 \times 10^{8}}{2 \times 10^{-4}} = 3 \times 10^{12}$

(3) $\frac{8 \times 10^{-5}}{2 \times 10^{-8}} = 4 \times 10^{3}$ (4) $\frac{8 \times 10^{-5}}{2 \times 10^{8}} = 4 \times 10^{-13}$

In these problems, the coefficients were intentionally made easy to handle. Try the examples below. This will give you needed practice in working with both coefficients and exponents where the sign conventions must be carefully observed.

Again, write out your answers to each of the examples. Which solution below is <u>incorrect</u>?

(12)

A This example is <u>incorrect</u>. $\frac{-4.8 \times 10^{-7}}{6.0 \times 10^{4}} = -8.0 \times 10^{-12}$

B This example is <u>incorrect</u>. $\frac{6.3 \times 10^{14}}{-9.0 \times 10^{-6}} = -7.0 \times 10^{19}$

C This example is <u>incorrect</u>. $\frac{-9.6 \times 10^{-6}}{-1.2 \times 10^{-9}} = 8.0 \times 10^{-3}$

CORRECT ANSWERS:

(1) 753 divided by 24.0 equals 3.14×10^1 or 31.4
(2) 24,300 divided by 0.0366 equals 6.64×10^5
(3) 0.000622 divided by 0.00622 equals 0.100 or 1.00×10^{-1}
(4) 2,730 divided by 2,700 equals 1.01
(5) 0.533 divided by 0.401 equals 1.33

NOTEBOOK CHECK

Refer to item 1(c) in your notebook. (Lesson 3)

Which of the following is an exact statement of this item?

(13)

A Move the decimal point in the coefficient to the left or right as necessary to position it properly.

B Round back the exponent to the proper number of significant figures.

C Round back the coefficient to the proper number of significant figures.

D Multiply the coefficients by ordinary arithmetic observing the rules for proper placement of the decimal point.

YOUR ANSWER --- B

You did not observe the entire rule. To determine the square root of a number expressed in scientific notation, you must find the square root of the coefficient and divide the exponent by 2.

You found the square roots of both the coefficient and the exponent.

Please return to page 27 and select another answer.

YOUR ANSWER --- B

Not bad, except for one thing. You did not pay attention to significant figures (but your arithmetic operations were correct). Note that the coefficient has 4 significant figures; hence the quotient coefficient must also have the same number.

It should be a cinch for you to choose the right answer now.

Please return to page 35 and select another answer.

YOUR ANSWER --- A

You're confused in the alteration of the power of ten.

The coefficient is 5.2948. This is larger than 5.0000, or more than 50% on its way toward the next higher power of ten. This indicates that you should multiply the existing power of ten (10^{-10}) by 10, and then drop the coefficient to express the order of magnitude.

What answer do you get for this operation?

$$(10^{-10}) \times 10^{1} = ?$$

Remember, you must be on the alert for peculiarities which are introduced by positive and negative exponents in combination. The fact that you selected 10^{-11} m as the order of magnitude shows that you were not giving attention to algebraic signs.

Please return to page 63 and select another answer.

YOUR ANSWER --- C

Incorrect. We have emphasized that you cannot compare orders of magnitude, that is, state how many times larger one is than the other, by a power of ten. This answer would say that the order of magnitude of one was 100 times (10^2) that of the other.

To determine the number of orders of magnitude greater (or smaller) one measurement is as compared with another, all you have to do is subtract the smaller from the larger exponent algebraically.

In this case, the correct operation is:

$$4 - (-2) = ???$$

Please return to page 64 and select another answer.

YOUR ANSWER --- C

Good work! You are correct. The <u>range</u> is defined as the total count of numbers between two extreme values in an ordered listing <u>without gaps</u>. But, if there are gaps -- as there are in TABLE 1 -- then a simple count will not do. The range in such listings is found by subtracting the smallest number from the largest number <u>algebraically</u> and adding 1. In TABLE 1, the orders of magnitude go from 10^{-22} all the way up to 10^{17} seconds. Working with the exponents:

$$17 - (-22) + 1 = \underline{40}$$

Thus, TABLE 1 has a range of 40 orders of magnitude.

Suppose you encountered a table showing an ordered listing of distances going from one of the following extremes to the other:

Distance to the farthest photographed galaxy = 10^{25} meters

Diameter of a proton = 10^{-15} meter

What would you say is the range of the orders of magnitude in this table?

Work it out, then turn to page 39 to check your answer.

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

YOUR ANSWER --- B

You are correct. To multiply powers of ten, you simply add the exponents and use this sum as the exponent in the product. Now let's take the next step. We want to multiply the following:

$$(5.6 \times 10^3) \times (2.1 \times 10^2)$$

In any series of sequential multiplications, such as this, you are always allowed to rearrange terms. Suppose we set it up this way:

$$5.6 \times 2.1 \times 10^3 \times 10^2$$

This doesn't change anything, does it? But we have already learned that $10^3 \times 10^2 = 10^5$ (adding exponents). Hence, we can write the example in this form:

$$5.6 \times 2.1 \times 10^5$$

Thus, the example resolves itself into a simple process in which the powers of ten are collected so that the exponents may be added, and the coefficients remain to be multiplied. What is the answer to this example?

(4)

A 11.76×10^5

B 1.176×10^5

YOUR ANSWER --- A

You are correct. The rule for multiplication of significant figures is: A product should contain the same number of significant figures as the least precise measurement. Since both measurements contain 2 significant figures, the product should have 2 as well.

The actual product, written in scientific notation, was 1.176×10^6 kg. Rounding back to 2 significant figures, the answer then is $\underline{1.2 \times 10^6 \text{ kg}}$. Here is another example, showing all steps separately:

$$(3.0 \times 10^2 \text{ cm}) \times (2.41 \times 10^4 \text{ cm})$$

$$\begin{aligned} &= 3.0 \text{ cm} \times 2.41 \text{ cm} \times 10^2 \times 10^4 \\ &= 3.0 \text{ cm} \times 2.41 \text{ cm} \times 10^6 \\ &= 7.230 \times 10^6 \text{ cm}^2 \\ &= 7.2 \times 10^6 \text{ cm}^2 \end{aligned}$$

Now try one for practice: A rectangle is 5.00×10^4 meters long and 1.11×10^5 meters wide. Find the area, to the correct number of significant figures.

(6)

A The area of this rectangle is 5.55×10^9 m^2.

B The area of this rectangle is 5.5500×10^9 m^2.

C The area of this rectangle is 555×10^7 m^2.

YOUR ANSWER --- A

You are correct. The mistake is easily spotted by working through the problem correctly:

$$6.100 \times 9.2 \times 10^{-3} \times 10^{8} = 56.1200 \times 10^{5}$$
$$= 5.61200 \times 10^{6}$$
$$\text{(to 2 significant figures)} = 5.6 \times 10^{6}$$

The correct exponent is "6" rather than the "5" given in the original solution.

Please go on to page 98.

NOTEBOOK ENTRY

Lesson 3

THE ARITHMETIC OF SCIENTIFIC NOTATION

1. Multiplication of numbers in scientific notation

(a) Multiply the coefficients by ordinary arithmetic, observing the rules for proper placement of the decimal point.

(b) Enter a "times" (x) sign after the coefficient product, then enter the product of the power of ten obtained by adding the exponents algebraically.

(c) Round back the coefficient to the proper number of significant figures.

(d) Move the decimal point in the coefficient product to the left or right as necessary to position it after the first non-zero digit, and correct the exponent to compensate for this movement, unless all significant figures can be written without the power of ten. A final answer may be written with or without the use of the power of ten, as long as the decimal point is located correctly. Thus, 3,542 may be written as 3.542×10^3.

Examples

$$(1.86 \times 10^5) \times (2.5 \times 10^{-2}) = 4.7 \times 10^3$$
$$(8.03 \times 10^{-12}) \times (6.12 \times 10^{-6}) = 4.91 \times 10^{-17}$$
$$(6.100 \times 10^{-3}) \times (9.2 \times 10^8) = 5.6 \times 10^6$$

Please go on to page 99.

We shall consider next a special case of multiplication. This is the process of squaring a number expressed in scientific notation. The square of a number is the number multiplied by itself. Thus, $3^2 = 9$, $5^2 = 25$, $8^2 = 64$, etc.

Since the process of squaring follows the rules of multiplication where the multiplier and multiplicand are the same number, we do not need a new set of instructions to perform the squaring operation.

For example: Square the measurement 186,000 mi/sec, considering it to have 3 significant figures.

Solution: Express the number in scientific notation.

$$(1.86 \times 10^5 \text{ mi/sec})^2$$

Square the coefficient; then square the power of ten. That is, $(10^5) \times (10^5) = 10^{10}$, obtained by adding exponents. This yields:

$$3.46 \times 10^{10} \text{ mi}^2/\text{sec}^2 \quad \text{(Answer.)}$$

Actually, the operation on the power of ten consists of doubling the exponents. Thus, to square a number in scientific notation, square the coefficient and double the exponent.

Try this example: Square the number 25,000 mi (approximate circumference of the earth). Work to three significant figures.

(8)

A 5.00×10^8 mi^2

B 5.00×10^{16} mi^2

C 6.25×10^8 mi^2

D 6.25×10^{16} mi^2

YOUR ANSWER --- B

You are correct. This is almost self-evident from the example.

Since $\frac{100,000}{100} = 1,000$, then $\frac{10^5}{10^2}$ also equals 1,000.

But $1,000 = 10^3$, so $\frac{10^5}{10^2} = 10^3$.

And 10^3 can only be obtained by subtracting the exponent of the denominator from the exponent of the numerator.

Now, let's see what happens when we have a coefficient. First, using standard notation, we shall divide 750,000 by 300.

$$\frac{750,000}{300} = 2,500$$

Doing the same thing in scientific notation, we obtain:

$$\frac{7.5 \times 10^5}{3 \times 10^2} = 2.5 \times 10^3 = 2,500$$

Right? Can you see the rule emerging from this example? Study the operations and then select the best answer from the following:

(11)

A In division, both the coefficients and the exponents are subtracted, denominator from numerator.

B In division, the coefficients are divided normally, while the exponents are subtracted, denominator from numerator.

C In division, the coefficients are divided normally; then the exponents are divided, denominator into numerator.

YOUR ANSWER --- A

You are correct. Since 10^{-3} carries an odd exponent, you shifted the decimal point in the coefficient one place to the right, obtaining $\sqrt{64 \times 10^{-4}}$. Then you found the square root of 64, or 8, and combined that with 10^{-2} to give you:

$$8.0 \times 10^{-2} = 0.08$$

It must have struck you that we have been using examples for practice in finding square roots in which the coefficients were perfect squares. To keep the work easy, we have avoided coefficients that would require a table of square roots, or that would require you to determine the square root by arithmetic methods. We shall continue to assume that you have a table of square roots or a slide rule handy, or that you know how to find square roots by standard methods.

Before we investigate addition and subtraction of numbers in scientific notation, we want you to have a short review of the methods developed thus far in this lesson. You will run through a few problems in which more than one operation is required. For example, perform the indicated operations in the problem below, giving attention to significant figures (3 significant figures are assumed throughout).

Set up this problem in scientific notation:

$$\frac{35{,}600 \times 0.00345}{2{,}720}$$

Please turn to page 83 to check your work.

YOUR ANSWER --- C

This arrangement is incorrect. You lined up all of the last digits with each other, without considering the respective place value of each of them.

The prime rule in adding columns of figures is that the unit column must form a straight vertical line, the tens column must form another straight vertical line, the hundreds column must form a third line, and so forth. You can be sure that the columns are correctly aligned by writing the numbers so that decimal points, written or implied, are directly below each other in a straight vertical line.

Please return to page 48 and select another answer.

YOUR ANSWER --- A

You should follow instructions carefully. You should change all the powers of ten to 10^4, which is the largest of the group. Then, you must also shift the decimal points wherever this change is made to compensate for the altered exponent. This means that your final arrangement prior to the actual addition should look like this:

$$\begin{array}{l} 0.0131 \times 10^4 \\ 8.45 \quad \times 10^4 \\ \underline{0.481 \quad \times 10^4} \end{array}$$

It would seem, from your answer, that you changed the powers of ten to 10^4 but did not shift the decimal point to compensate for these changes.

Please return to page 75 and select the correct answer.

YOUR ANSWER --- D

This example is not correctly handled. We'll run through the operations carefully because we have something important to point out in this particular case.

Converting: Bringing 10^{-2} up to 10^{2} requires that we multiply 10^{-2} by 10^{4}. Thus:

$$6.43 \times 10^{-2} = 0.000643 \times 10^{2}$$

Subtracting:

$$\begin{array}{r} 1.38 \quad\quad\;\; \times 10^{2} \\ -\,0.000643 \times 10^{2} \\ \hline 1.379357 \times 10^{2} \end{array}$$

Rounding back: $1.379357 \times 10^{2} = \underline{1.38 \times 10^{2}}$

You may be startled to find that the difference comes out exactly the same as the larger of the two original numbers. Clearly, the quantity 0.000643×10^{2} is insignificant compared to 1.38×10^{2}, hence cannot affect the result of 3 significant figures. This situation is frequently encountered in addition and subtraction; you must be on your guard for it. Generally, in addition or subtraction, when one power of ten is 10^{4} times as large as another, then the smaller quantity cannot affect the larger quantity in problems limited to 3 significant figures.

Please return to page 36 and select another answer.

YOUR ANSWER --- B

Sorry! You're not following the rules.

Look at the coefficient. It is 5.2948. Isn't this larger than 5.0000? Since it is larger, should the exponent of the power of ten be allowed to remain (-10)?

When the coefficient is larger than 5.0000, as in this example, the power of ten is multiplied by 10 and the coefficient is dropped to express the order of magnitude. What is the value of $(10^{-10}) \times 10^{1}$? This will give you the right answer.

Please return to page 63 and select another answer.

YOUR ANSWER --- B

Not quite right. You subtracted the smallest exponent (10^{-22}) from the largest exponent (10^{17}) algebraically. This just misses the right answer. Note this list:

3
4
5
6
7
8

The range is defined as the total count of numbers between two extreme values in an ordered listing without gaps, <u>including both extremes</u>. In this list, you can count 6 numbers <u>including</u> the extremes; hence the range is 6. Or, you may subtract algebraically and add 1. Thus, $8 - 3 + 1 = 6$.

Now consider a second example:

3
2
1
0
-1
-2
-3

In this case, the range is 7. By actual count there are 7 numbers including the extremes. Or, subtracting algebraically and adding 1, we have: $3 - (-3) + 1 = 7$.

Thus, the range of an ordered progression of numbers with or without gaps may be obtained by subtracting the smallest from the largest and adding 1, <u>whether or not there is a zero included in the list</u>.

In selecting your answer, you subtracted algebraically correctly but did not add 1. As shown above, this extra "1" is necessary.

Please return to page 24 and select another answer.

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YOUR ANSWER ---- A

You've forgotten your terminology. We have never used the phrase "main number" to describe anything we've discussed thus far.

Please return to page 3 and select another answer.

WORKSHEET

Please listen to Tape Segment 1 of Lesson 3 before starting this Worksheet.

Always select answers for Worksheet questions by punching them out on the special AV Computer Card.

QUESTIONS

1. According to the lecturer who presented this portion of the audio tape, which one of the following statements is most valid?

   A Knowledge of arithmetic is unimportant in the study of physics.
   B One does not have to have a mind like a high speed computer to become a physicist.
   C There are some perfectly normal people who can never learn to add even two simple whole numbers.
   D Everyone starts out in life with the same latent ability to handle arithmetic, but some become more proficient than others because they practice.
   E A grocer who can add a long column of figures accurately and quickly must have a high intelligence level.

2. Understanding and applying concepts in physics requires

   A that you know how to use a slide rule.
   B that you ultimately become familiar with electronic computers.
   C that you learn how to use tables of squares and square roots.
   D that you develop reasonable accuracy in handling arithmetic.
   E none of the above.

3. A famous scientist is said to have had trouble with high school geometry. Which one is it?

   A Albert Michelson.
   B Robert Oppenheimer.
   C Isaac Newton.
   D Albert Einstein.
   E Galileo Galilei.

When you have finished punching out your answers to the above questions, please return to page 2 of the Study Guide.

WORKSHEET

Please listen to Tape Segment 2 of Lesson 3 before starting this Worksheet. Always select answers for Worksheet questions by punching them out on the special AV Computer Card.

DATA ITEM A: The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.

DATA ITEM B: $(a \times 10^b)^n = a^n \times 10^{bn}$.

DATA ITEM C: $(2.0 \times 10^2)^4 = 2.0^4 \times 10^{2 \times 4} = 16 \times 10^8 = 1.6 \times 10^9$.

DATA ITEM D: $(2.0 \times 10^2)^4 = 200^4$

but $200 \times 200 = 40{,}000$ (This 200 squared)

and $200 \times 40{,}000 = 8{,}000{,}000$ (This is 200 cubed)

and $200 \times 8{,}000{,}000 = 1{,}600{,}000{,}000$ (This is the 4th power)

so $(2.0 \times 10^2)^4 = 1.6 \times 10^9$ which is what we set out to prove.

QUESTIONS

4. If the radius of a sphere is given in "zilches", then V will come out in

   A zilches.
   B square zilches.
   C cubic zilches.
   D cubic inches, if 1 zilch = 1 ft.
   E cubic ft, if 1 zilch = 1 inch.

5. Which one of the following is equivalent to $(z \times y^a)^b$?

   A $z^b \times y^{ab}$
   B $z^a \times y^{ba}$
   C $z^b \times y^b$
   D $z^a \times y^b$
   E $z^b \times y^a$

6. With proper attention to significant digits, which one of the following is the correct expansion of $(4.20 \times 10^4)^2$?

   A 17.6×10^{15}.
   B 17.64×10^8.
   C 17.64×10^{15}.
   D 17.64×10^2.
   E 17.6×10^8.

When finished, please return to page 44 of the Study Guide.

WORKSHEET

Please listen to Tape Segment 3 of Lesson 3 before starting this Worksheet. Always select answers for Worksheet questions by punching them out on the special AV Computer Card.

DATA ITEM A: $\sqrt{n} = n^{1/2}$

DATA ITEM B: $\sqrt{25} \times \sqrt{25} = ?$

$5 \times 5 = 25$

$\sqrt{n} \times \sqrt{n} = n$ or n^1

$n^? \times n^? = n^1$

$n^{1/2} \times n^{1/2} = n^1$ THEREFORE: $\sqrt{n} = n^{1/2}$

QUESTIONS

7. Using the fractional method of power notation, how would you write the cube root of the quantity $(n + 2)$?

   A $3\sqrt{n} + (2)^{1/2}$
   B $(n)^2 + 1/3$
   C $(n + 2)^{3/2}$
   D $(n + 2)^{1/3}$
   E $n^{1/3} + 2^{1/3}$

8. Using the fractional method of power notation, write the fifth root of Z.

   A Z^5
   B Z^{-5}
   C $Z^{1/5}$
   D $Z/5$
   E none of these is correct.

9. What is the product of $(25)^{1/2}$ times $(81)^{1/2}$?

   A 2025
   B 45
   C 225
   D 405
   E 20.25

When finished, please return to page 48 of the STUDY GUIDE.

WORKSHEET

Please listen to Tape Segment 4 of Lesson 3 before starting this Worksheet. Always select answers for Worksheet questions by punching them out on the special AV Computer Card.

DATA ITEM A: Length of tape = 12.64 meters (Tape 1)
Length of tape = 12.62 meters (Tape 2)

difference = 0.02 meter = 2×10^{-2} m = 2 cm

QUESTIONS

10. Which one of the following processes should, if possible, be avoided when working with numbers where significant digits are important?

    A Subtraction of two numbers of nearly equal magnitude.
    B Subtraction of a small number from a large number.
    C Addition of two numbers of nearly equal magnitude.
    D Subtraction of a large number from a small number.
    E Addition of two very small numbers.

11. If the difference between two length measurements is desired when the two lengths to be measured are nearly the same, it is a good practice to

    A ignore significant digits completely
    B subtract as usual but then add enough digits to insure the correct number of significant figures.
    C subtract as usual but then move the decimal point to include the correct number of significant digits.
    D place enough zeros in front of the first non-zero digit so that the answer has the correct number of significant digits.
    E place the two objects side by side and measure the difference in length directly if possible.

When finished, please return to page 36 of the STUDY GUIDE.

WORKSHEET

Please listen to Tape Segment 5 of Lesson 3 before starting this Worksheet. Always select answers for Worksheet questions by punching them out on the special AV Computer Card.

QUESTIONS

12. Which one of the following was NOT mentioned in the tape discussion to which you have just listened?

A The difference in order of magnitude of a day compared to the time for one turn of a fan.
B Fossils help us determine the age of a geological formation.
C A discussion of the method used by physicists to to measure the time required for a single spin of a proton on its axis.
D We can measure the age of the earth by methods which involve radioactivity.
E Stating the order of magnitude of a certain interval may be more meaningful than giving the interval itself especially if the latter is variable.

13. For time intervals having orders of magnitude below 10^{-10} second, measurements have to be made

A by means of microsecond timers.
B vibrations of subatomic particles.
C electrical stopwatches.
D radio waves.
E none of these is correct.

14. According to Table 1 in the STUDY GUIDE, it takes about 10 sec to wind your wristwatch. Suppose you found that it required 20 sec to wind a grandfather clock. What is the difference in order of magnitude of these two intervals?

A One order of magnitude.
B Two orders of magnitude.
C Negative-one order of magnitude.
D No difference in orders of magnitude.
E The difference cannot be expressed in orders of magnitude.

When finished, please return to page 11 of the STUDY GUIDE.

PROBLEM ASSIGNMENT

Articulated Multimedia Physics

LESSON 3

Instructions: Refer to head of PROBLEM ASSIGNMENT for LESSON 1.

1. Multiply the following:

   (a) $(2 \times 10^7) \times (3 \times 10^4)$ (b) $(4 \times 10^2)(2 \times 10^3)$

   (c) $(4 \times 10^{-2})(2 \times 10^{-3})$ (d) $(3 \times 10^5)(2 \times 10^{-3})$

2. Solve the following:

   (a) $\sqrt{81 \times 10^8}$ (b) $\sqrt{36 \times 10^{-4}}$

   (c) $\sqrt{22.5 \times 10^7}$ (d) $\sqrt{1.21 \times 10^{-6}}$

3. Square each of the following expressions:

   (a) 5.1×10^5 (b) 3.05×10^3

   (c) 4.110×10^{-9} (d) 8.00×10^{-2}

4. Perform the following divisions:

   (a) $8 \times 10^6/2 \times 10^4$ (b) $(9 \times 10^{-2})/(3 \times 10^4)$

   (c) $(2.4 \times 10^{-6})/(3 \times 10^{-2})$ (d) $(2.4 \times 10^{-6})/(3 \times 10^{-12})$

5. Perform the indicated additions:

   (a) $4 \times 10^2 + 3 \times 10^2$ (b) $4.1 \times 10^3 + 3.0 \times 10^2$

   (c) $4.0 \times 10^{-3} + 3.1 \times 10^{-2}$

6. What isthe order of magnitude of each of the following:

   (a) 3.21×10^{10} (b) 7.5×10^4

   (c) 4.99×10^6 (d) 11,000

7. What is the difference in order of magnitude between 4.9×10^{-7} and 5.1×10^8?

8. Find the cube of 2.2×10^{-4} with the proper number of significant digits.

9. Write each of the following using fractional notation for roots.

(a) The square root of K.

(b) The cube root of A + B + C

(c) The twelfth root of 8,746,000

(d) The fifth root of Z/Y.